Performance of Joint Multiuser Detection and Group Decoding for Trellis Coded DS-CDMA Systems

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Abstract—Optimal multiuser detection of trellis coded signals has prohibitive complexity. A suboptimal concatenated scheme consisting of a maximum a posteriori (MAP) multiuser interference (MUI) detector followed by a deinterleaver and a soft trellis decoder has recently received considerable attention in the literature [1]. We call this scheme Concatenated Multiuser Detector and Trellis Decoder (CMDTD). An alternative to CMDTD is Joint Multiuser Detection and Group Decoding (JMDGD), introduced in [2] and [3]. In JMDGD, reduced-state sequence estimation (RSSE) is used to jointly decode the coding information of a group of users and equalize the MUI.

In this paper we show that JMDGD outperforms CMDTD in asynchronous trellis-coded DS-CDMA systems over high-interference AWGN channels. In these channels, the advantage of JMDGD is dramatic in near-far situations. Furthermore, the performance edge of JMDGD over CMDTD increases as the MUI increases. In fast fading channels, simulation results show that JMDGD performs close to CMDTD with significantly reduced complexity.

I. INTRODUCTION

Performance of CDMA systems is greatly enhanced by the use of trellis coding. Since the complexity of optimal joint MUI detection and trellis decoding is prohibitive, suboptimal reduced complexity receivers that perform multiuser detection and decoding disjointly have been proposed. Among them, CMDTD has received considerable attention [1]. The multiuser detector in CMDTD provides soft outputs in the form of symbol-by-symbol a posteriori probabilities. Although soft outputs from multiuser detection and ideal interleaving enable users’ decoders to reach optimum performance, the code information is not exploited to improve MUI equalization. Recently, reduced complexity receivers based on JMDGD have been proposed in [2] and in [3]. A comparative study of performance and complexity of these two schemes is presented in this paper.

The only way a partitioned scheme such as CMDTD can benefit from information derived by the users’ decoders is by iterative processing. Performance of JMDGD can also be improved by iterative processing. However, in this work we focus on the improvements achieved by JMDGD over the CMDTD in non-iterative mode.

Reduced complexity receivers based on JMDGD were first presented in [2] and [3]. In [3] a reduced complexity receiver for convolutionally coded synchronous multiple-access channels is proposed. This receiver, called Group Metric (GM) decoder, decodes a subset of users and exploits all the multiuser information in its metric. Unlike the synchronous multiuser channel model, the asynchronous model must account for the fact that the users transmit streams of symbols, therefore, this receiver needs extra processing to deal with multiuser interference. A reduced complexity receiver for asynchronous channels with trellis coding is proposed in [2]. Based on the coding information of a subset of users and the set partitioning principles inherent in trellis coded modulation, reduced state trellises are constructed. This receiver uses a delayed decision sequence estimation algorithm to jointly decode the given subset of users and equalize all the multiuser interference. For synchronous channels, this receiver reduces to the GM decoder. In [2] and [4] the structure that jointly decodes one user and equalizes the multiuser interference is analyzed.

In this work we investigate the performance of JMDGD in asynchronous trellis-coded DS-CDMA systems over AWGN and Rayleigh fading channels. We compare the performance and complexity of CMDTD and JMDGD receivers, and present simulation results in situations of practical interest such as high bandwidth efficiency situations. We show that JMDGD can perform better than CMDTD with significantly reduced complexity. The advantage of JMDGD is dramatic in near-far situations over AWGN channels. Furthermore, the performance edge of JMDGD over CMDTD increases as the MUI increases.

The rest of the paper is organized as follows. In Section 2, the discrete-time model for asynchronous CDMA is presented. Section 3 describes the JMDGD scheme. The complexity of JMDGD and CMDTD is analyzed in Section 4. In Section 5, the performance of these receivers is studied and their complexities are compared. Finally concluding remarks are given in Section 6.

II. SYSTEM MODEL

Consider a trellis coded asynchronous CDMA system with $K$ users. Each user employs trellis coding. The baseband received signal can be written as

$$r(t) = \sum_{i=-\infty}^{\infty} \sum_{k=1}^{K} h_k c_k(i) s_k(t - iT - \tau_k) + v(t), \quad (1)$$

where
... additive colored Gaussian noise is at the output of the matched filters. The spectrum of the additive sequence, the transmitted sequence, and the noise sequence are the vector-valued z-transforms of the matched filter output, while 

\[ z \in \mathbb{C} \] represents the zero mean colored Gaussian noise vector at the filter output, while \( G[0], G[1], \) and \( G[-1] \) are the cross-covariance matrices of users. The total number of states necessary to decode the \( m \) users is \( \sum_{i=0}^{K} \mathbf{F}_{i} \), where \( K \) is the fixed decision delay. In addition, the MAP algorithm requires \( O_{MAP} = M_{MAP} * A_{\Gamma} \) operations. To avoid unfairly penalizing the complexity of CMDTD, we assume a slightly suboptimal approximation to the MAP algorithm that eliminates a large number of exponentiations and multiplications [5].

On the other hand, with TBCED(1), \( K \) receivers are needed to decode the \( K \) users. For \( m \) larger than 1, \( G \) parallel receivers (each of them using the coding information of \( m_{s} \) users) decode the \( K \) users. The total number of states necessary to decode the \( K \) users is given by

\[ S_{TBCED(m)} = \sum_{g=1}^{G} S_{g} S_{m_{s}} \].

### III. JOINT MULTIUSER DETECTION AND GROUP DECODING

Let \( y_{k}(i) \) be the output of the discrete-time white noise model of the \( k^{th} \) user at instant \( i \). The maximum likelihood problem for the asynchronous coded CDMA white noise channel can be stated as follows: find the coded sequence \( \{ \hat{c}_{k}(i) \} \) that minimizes the cumulative metric

\[ M = \sum_{i=0}^{M} \sum_{k=1}^{K} \frac{k(i) - \sum_{i=1}^{k} F_{ik} h_{k}(i) \hat{c}_{k}(i) - \sum_{i=k+1}^{K} F_{k} h_{k}(i) \hat{c}_{k}(i) - 1}{\|} \]  

Assuming that each user encoder has \( S \) states, from (7) we verify that the optimum super-trellis requires

\[ S_{opt} = S^{K} 2^{(K-1)} \]  

states [2], which is prohibitive in practical implementations.

To reduce the number of states, in [2] a suboptimal scheme is proposed that uses the coding information of a subset of users and the information of the multiuser channel. The scheme decodes a subset of \( m \) users (with \( m \leq K \)) and equalizes the MUI of the channel. The number of channel states is reduced applying the set partitioning principles inherent in TCM. This receiver is called trellis-based combined multiuser interference equalization with \( m \)-user decoding, TBCED(\( m \)). For synchronous multiuser systems, this receiver reduces to the Group Metric receiver proposed in [3] for convolutionally coded channels. Examples of this reduced complexity receiver were presented in [4] for several reduced-state multiuser receivers.

### IV. COMPLEXITY ANALYSIS

We now analyze the complexity of the TBCED(\( m \)) and that of CMDTD. The complexity of CMDTD is dominated by the MAP detector. As we consider continuous-mode detection with fixed decision delay, in this work we analyze the type-II MAP algorithm that requires only a forward recursion [5]. The complexity comparison is based on the requirements of memory units and the number of operations per symbol. The memory requirements for the MAP algorithm are

\[ M_{MAP} = |X| + |X| (D_{MAP} - K + 1)(Q - 1) \]  

where \( |X| = Q^{K-1} \) is the number of states, and \( D_{MAP} > K - 1 \) is the fixed decision delay. In addition, the MAP algorithm requires

\[ O_{MAP} = M_{MAP} * A_{\Gamma} \]  

operations, where \( A_{\Gamma} \) operations are needed to compute the branch metric \( |X| \) multiplication and \( |X| - 1 \) additions. To avoid unfairly penalizing the complexity of CMDTD, we assume a slightly suboptimal approximation to the MAP algorithm that eliminates a large number of exponentiations and multiplications [5].

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\[ S_{TBCED(m)} = \sum_{g=1}^{G} S_{g} S_{m_{s}} \].
where $S_{g}$ is the number of channel states in the $g^{th}$ parallel receiver (it depends on the level of subset partitioning used for each symbol of the multiuser channel), and $S$ is the number of states of the user’s encoder. The memory requirements for the TBCED($m$) are

$$M_{TBCED(m)} = S_{TBCED(m)} \times (K + D_{V}),$$

(12)

where $D_{V}$ is the decision depth of the conventional Viterbi decoder. For each decoded symbol, the Viterbi algorithm makes $K$ transitions, hence the number of operation required per decoded symbol is

$$O_{TBCED(m)} = S_{TBCED(m)} \times K \times A_{F}.$$  

(13)

The performance and complexity of the TBCED are controlled by the number of states $S_{TBCED}$. In the next section we compare the performance and complexity of different TBCED with those of the CMDTD. Note that the number of operations required by TBCED depends on the total number of states, while the number of operations required by MAP depends on the memory size.

V. NUMERICAL RESULTS

The objective of this section is to study the performance of TBCED and CMDTD in bandwidth-efficient CDMA environments and in near-far situations, and compare their complexities. We simulate a four user asynchronous DS-CDMA system. Each user employs a four-state convolutional encoder followed by a signal mapper that uses a 4-ary signal constellation (Fig. 1).

We consider a short Gold spreading sequence with 7 chips/coded-symbol. The relative delays of users are constant for the simulation. The multiuser channel spectrum is given by

$$S(D) = \frac{1}{7} \left[ \begin{array}{cccc} 7 & -4 - D & 1 + 2D & -D \\ -4 - D^{-1} & 7 & -4 - D & 3 \\ 1 + 2D^{-1} & -4 - D^{-1} & 7 & -2 + D \\ -D^{-1} & -2 + D^{-1} & 7 & 7 \end{array} \right].$$

(14)

TBCED receivers with 32 states are considered along with the CMDTD that uses a forward algorithm to implement MAP detection [5] and a block interleaver size of $(30,30)$. The memory and operations required by the individual decoders in the CMDTD scheme are ignored in the complexity comparison. The TBCED(1,32) (in slight departure from our previous notation, we call TBCED($m$, $s$) the receiver that decodes $m$ users with $s$ states) receiver uses the coding information of only one user and an 8-state MUI channel $(4 \times 8 = 32)$, while the TBCED(2,32) receiver uses the coding information of two users and only a 2-state MUI channel $(4^{2} \times 2 = 32)$. TBCED(2,32) decodes two users simultaneously, therefore two TBCED(2,32) are sufficient to decode four users. However, if the interference comes mainly from one of the users, three TBCED(2,32) may be required to achieve high performance gains. The complexity comparison, obtained in the previous section, is shown in Table I for a fixed decision delay in the MAP detector equal to five times the memory of the channel (i.e. $D_{MAP}=15$), and a decision depth in the TBCED equal to five times the memory of the trellis code (i.e. $D_{V}=10$). In this case three TBCED(2,32) are used to decode the four users. This corresponds to the worst case.

First we analyze the performance of these receivers over an AWGN channel. Simulation results for equal power users are shown in Fig. 2. The BER of the four users are geometrically averaged. At BER=$10^{-4}$, the CMDTD performs a little better than the TBCED(1,32), but the latter requires about 2 times less memory and 6 times fewer operations. Using three TBCED(2,32) to decode the four users, the performance improves by 0.5 dB over the TBCED(1,32) and 0.2 dB over CMDTD. TBCED(2,32) requires 2.4 times less memory and 8 times fewer operations than CMDTD. This result shows that coding information is more effective than multiuser channel state information to improve performance with JMDGD schemes.

Fig. 3 shows simulation results for a near-far situation. The SNR of two of the users is held constant at 4 dB, while that of the rest of the users is increased. The TBCED(2,32) uses the coding information of the two users with the lowest power. The performance of the two schemes improves as the interference from the other two users grows stronger. This phenomenon is similar to that observed in the successive interference cancellation detector. The imbalance in power levels increases the correct detection probability for the high power users. Thus, the detected signals can be used to cancel successfully the interference, increasing the correct decoding probability for the low power users. TBCED(2,32) performs 2.5 dB better than CMDTD at BER=$3 \times 10^{-3}$. Quite interestingly, it is noted that the performance gap between the TBCED(2,32) and CMDTD increases as the SNRs of the interfering users increase (which results in increased MUI affecting particularly the low power users). This is because the code is more effective to combat error propagation as the MUI estimation improves.

Fig. 4 shows the performance of these receivers over a very fast fading channel. We consider a flat independent Rayleigh fading model. The diversity gain of a trellis code in an independent fading channel is given by the Hamming distance of the shortest error event path [6]. The Hamming distance of the trellis code shown in Fig. 1 is 5. Note that all receivers achieve the diversity gain of the code. This is coincident with the result obtained by Caire in [7] who shows that the asymptotic order (i.e., the slope of the BER curve versus SNR) of a maximum
likelihood receiver is equal to the asymptotic order of the conventional receiver in MUI free transmission. Fig. 4 shows that the performance of TBCED(1,32) is almost the same as that of TBCED(2,32), which is close to the CMDTD. Fig. 4 also shows the performance of the single user channel. It is shown in this figure that the analyzed receivers perform near the single user bound.

In very slow fading channels, trellis codes without interleavers do not get diversity gain. In these channels, one of the most effective diversity schemes is space-time coding. The JMDGD scheme for space time trellis codes is studied in [8], where it is shown that JMDGD performs near the single user bound. In this situation, the number of states of the MAP detector is given by $S = Q^{N(K−1)}$, where $N$ is the number of transmitter antennas. To get the maximum diversity gain with minimum complexity, the number of states of a space time trellis code is equal to $Q$ [9]. With this code, the JMDGD that uses the coding information of all the users and ignores the multiuser channel states has $Q^K$ states. Therefore, if $K > (N/N_0 − 1)$, then the number of states of a JMDGD that uses the coding information of all users is smaller than that of a MAP detector.

VI. CONCLUSIONS

A JMDGD receiver has been proposed for heavily loaded asynchronous CDMA systems over AWGN and very fast fading channels. It is observed that JMDGD can perform close to the CMDTD scheme with significantly reduced complexity. In near-far situations over AWGN channels, the performance of JMDGD improves dramatically over that of CMDTD. The performance of these two schemes can be further improved by iterative detection. Future directions for research includes iterative receivers with JMDGD subsystems.

REFERENCES


