Symbol Merging Approach for Intercell Interference Mitigation in Wireless OFDM Systems

Enrique Mariano Lizarraga∗, Alexis Alfredo Dowhuszko†, and Victor Hugo Sauchelli‡
∗Digital Communications Research Laboratory, National University of Cordoba - National University of Catamarca - CONICET. Av. Velez Sarsfield 1611, Cordoba (X5016GCA), Argentina. Email: emlizarra@conicet.gov.ar
†Department of Communications and Networking (Comnet), Aalto University
P.O. Box 13000, FI-00076 Aalto, Finland. Email: alexis.dowhuszko@aalto.fi
‡Research Laboratory of Mathematics Applied to Control, National University of Cordoba
Av. Velez Sarsfield 1611, Cordoba (X5016GCA), Argentina. Email: vsauch@com.uncor.edu

Abstract—Out-of-band power emissions impose stringent limits on the performance that new techniques, such as intercell interference coordination (ICIC) and cognitive radio (CR), provide over wireless systems with OFDM-based air interfaces. To tackle this problem, in this work we consider the use of a simple symbol merging (SM) approach, to enable the reduction of adjacent channel interference when generating the OFDM signal. The basic idea behind this proposal is simple: implement an inverse discrete Fourier transform (IDFT) calculation with double length, to merge OFDM symbols in pairs and reduce by half the number of points with discontinuities in time domain (compared to a conventional OFDM transmission). In addition, a spectral precoding technique is also introduced, to provide a continuous time behavior in the concatenation points that remain after the merging. Based on different performance analyses it is possible to conclude that, our proposed spectral precoding method guarantees a reduced distortion level in the frequency domain, when compared to traditional N-continuous OFDM techniques reported in the literature. Finally, the effect of avoiding the use of cyclic prefix (CP) in part of the OFDM symbols is also studied.

I. INTRODUCTION

The number of wireless-enabled devices [1] and the amount of data per mobile subscriber [2] have been increasing constantly during the last decade. Since this tendency is expected to continue (and even grow) in the next few years, a non-myopic approach will be required in the future for wireless systems that are designed to cope with this demand. In other words, next generation wireless systems will not only necessitate improved spectral efficiencies to achieve their claimed gains, but they will also require additional portions of radio spectrum (released for wireless usage) and denser deployment of small cell base stations in hot spot areas (for improved frequency re-usage) [3]. The benefits of these three approaches go together hand in hand; therefore, a joint combination of them within an OFDM-based air interface is expected to be the main candidate to meet these requirements in the future. As expected, the limiting factor in all these cases is the interference strength observed in reception [4]. Therefore, a key parameter that should be definitely kept under control is the out-of-band power emission that OFDM-based systems generate in transmission.

Cognitive radio (CR) is a technology that enables the provision of additional frequency bands, e.g., implementing dynamic spectrum access (DSA) to license-exempt secondary users whenever reception in licensed primary users is not (considerably) affected. For example, CR technology allows to exploit so-called TV white spaces around the world, as TV broadcast services in VHF/UHF bands switch from analog to digital. The basic idea in this case is to allow secondary devices to operate on vacant TV channels, whenever no harmful interference is generated to adjacent primary TV channels [5]. Note that the level of out-of-band power emissions, generated by secondary transmissions, becomes a critical factor to capitalize the gain advertised by this proposal. Similarly, the denser deployment of base stations in both, macro- and femto-layer, suggests the use of fractional frequency reuse (FFR) schemes to keep under control the co-channel interference in an heterogeneous network (using different degrees of reuse factors within the same macro-cell) [6]. As expected, the effectiveness of FFR will depend on the ability of transmitters to keep the OFDM signal confined within the desired partitions of the channel. Again, the reduction of out-of-band power in OFDM transmissions plays a key role in the performance of a given FFR pattern, proposed for (enhanced) intercell interference coordination (ICIC) in future wireless networks.

An OFDM signal is a sequence of OFDM symbols, each one consisting of a collection of modulated orthogonal subcarriers. Since the amplitudes and phases of these subcarriers are often statistically independent, OFDM symbols are commonly assumed to be independent as well. So, the concatenation of OFDM symbols generates discontinuities in the time domain signal, or equivalently, high power emissions out of the desired band. Current standards use filtering techniques to limit the power level of these undesired out-of-band emissions; nevertheless, the main drawback of this approach is a reduction in the effectiveness of the CP [7]. Another option to control this problem is to implement adaptive symbol transition [8]. However, the pitfall in this case is that since transmitter processing needs to be updated in every symbol, the complexity of the system is required to be increased as well.

This work focuses on the generation of OFDM signals that comply with quality requirements, frequently specified by emission masks, such as the ones that are presented in LTE specifications [9]. In other words, we address the generation of an oversampled digital signal that improves the spectral allocation of power in both, in-band and out-of-band regions. The key idea behind the proposed scheme is to correlate
two OFDM symbols in the frequency domain, to obtain an equivalent continuous response in the time domain signal. After this, the concatenation points of the OFDM symbols will show a continuous behavior in half of the original cases (that correspond for the whole OFDM transmission). A preliminary version of this merging concept was previously presented in [10].

In addition, a spectral precoding technique is also introduced to guarantee a continuous time-domain response in those concatenation points that remain after the merging process. Unlike [8], this technique is a static precoding procedure, and does not demand extra bandwidth like in alternative proposals [11]. Simulation results show an important reduction in the error-vector magnitude (EVM) as performance measure. The effect of skipping the insertion of CP in part of the OFDM symbols is also considered, obtaining an additional reduction in out-of-band emissions with an improved usage of channel resources. This latter concept is comparable to the one presented in [13], but in our case structural differences are observed in the scheme that is proposed for decoding.

The rest of this paper is organized as follows. Section II presents the OFDM signal equations that are used throughout this work. The fundamentals of our proposed merging strategy and our reduced distortion precoding technique are presented in Section III and Section IV, respectively. Section V covers the channel modeling and the system operation at the receiver side, while simulation results are given in Section VI. Finally, Section VII shows the conclusions.

Notation: Vectors are indicated in bold and lower-case letters, unlike complex or real scalar numbers. Matrices are indicated in bold with capital letters. Superscripts \((*)^T\), \((*)^H\), and \((*)^{-1}\) represent transpose, Hermitian, and inversion, respectively. Notation \(0_{M \times N}\) indicates the \(M \times N\) matrix with all-zero entries, while \(I_M\) stands for the \(M \times M\) identity matrix. Convolution is noted by the * operator.

II. SYSTEM MODEL

The OFDM transmitter is supplied by a bitstream. This bitstream is mapped by a conventional complex modulator (e.g., M-QAM, M-PSK, etc.), and its output is collected by a serial-to-parallel block in a \(K\)-element vector to conform the \(i\)-th OFDM symbol in frequency domain \(d_i\). An OFDM symbol in time domain is obtained by means of an inverse fast Fourier transform (IFFT) calculation \(s_i = \text{IFFT}(d_i, K)\), where IFFT\((a, b)\) represents the IFFT calculation with \(b\) points applied on the \(a\) vector. The vector \(s_i\) is related to the continuous time symbol \(s_i(t)\), with symbol period \(T_s\) and guard interval \(T_g\). This procedure implies a so-called subcarrier allocation scheme \(\mathcal{K}\), which controls the set of frequencies that are used; in this case, \(\mathcal{K} = \{-K/2, ..., -1, 1, ..., K/2\}\). Besides, several standards define balanced subcarrier allocation schemes, e.g.,

\[
\mathcal{K} = \{-K/2, ..., -1, 1, ..., K/2\},
\]

which is included in [14]. To comply with this subcarrier mapping, \(u^K_i = \text{CSH}(d^K_i, \frac{K}{2})\) feeds the IFFT block, where \(d^K = (d_{i,0}, ..., d_{i,K/2-1}, 0, d_{i,K/2}, ..., d_{i,K-1})^T\) and CSH\((a, b)\) represents a circular up-shift operation of \(b\) positions in the column vector \(a\).

In applications like software defined radio (SDR) or CR, the bandwidth that is considered to generate the output signal is allowed to be larger than the bandwidth assigned in the wireless channel. The aim in this case is to have power emissions adequately accommodated within a certain mask (that is not necessarily fixed). To achieve this goal, an oversampled discrete-time output signal is required. Oversampling is related to interpolation, and it does not demand extra bandwidth resources for communication purposes. Let us define \(\eta\) as the oversampling factor. Then, an oversampled discrete-time output signal is \(s^K_i = \eta \text{ IFFT}(u^K_i, (K+1)\eta)\), where \(u^K_i\) can be treated as a conventional zero-padded version of \(u^K_i\), defined by \(u^K_i = \begin{pmatrix} u_{i,0}^K, ..., u_{i,K/2}^K, 0_{1 \times (K+1)(\eta-1)}, u_{i,K/2+1}^K, ..., u_{i,K}^K \end{pmatrix}^T\).

At this stage, \(s^K_i\) attains the form of a vector with \((K+1)\eta\) entries, containing the oversampled output signal.

The classical CP insertion can be performed with extension \(s^N_{i,K} = (s^N_{i,0}, ..., s^N_{i,(K+1)\eta-1})^T\), where \(\nu = \frac{T_g}{T_s}\) represents the CP fraction (i.e., the guard interval fraction). An OFDM symbol with CP is then obtained by concatenation, and is given by \(s^N_{i,K} = (s^K_i, s^K_{i,(K+1)\eta})^T\). The non-oversampled expression (i.e., \(s^K_i\)) is stated by setting \(\eta = 1\).

III. SYMBOL Merging Scheme

Our proposal is based on a symbol merging (SM) scheme, where \(d_i\) and \(d_{i+1}\) are correlated for \(i\) even (i.e., \(i = 0, 2, \ldots\)). Taking into account a transmitter with subcarrier allocation (1), we define

\[
\begin{align*}
\Delta^{K,+} &= \frac{1}{\sqrt{2}} (d^K + d^{K+1}_i) \\
\Delta^{K,-} &= \frac{1}{\sqrt{2}} (d^K - d^{K+1}_i)
\end{align*}
\]

for \(i\) even, and pair \(u^{K,+} = \text{CSH}(d^{K,+}_i, \frac{K}{2})\) and \(u^{K,-} = \text{CSH}(d^{K,-}_i, \frac{K}{2})\) as its corresponding shifted versions. Then, the time domain OFDM symbols are derived from

\[
\begin{align*}
s^{K}_i &= \frac{1}{\sqrt{2}} \text{IFFT} \left[ u^{K,+}, K+1 \right] \\
s^{K}_{i+1} &= \frac{1}{\sqrt{2}} \text{IFFT} \left[ u^{K,-}, K+1 \right]
\end{align*}
\]

These statements represent a simplified alternative, compared to the one presented in [10]. In addition, this new approach allows a receiver operation quite similar to the conventional one. Meanwhile, in accordance with the analysis presented in [10], it is possible to see that the concatenation of \(s^{K}_i\) with \(s^{K}_{i+1}\) can be related to a double-length inverse discrete Fourier transform (DFT) calculation

\[
s^{K'} = (s^{K}_{i}, s^{K}_{i+1})^T = \text{IFFT} \left[ u^{K,+}, 2(K+1) \right] \]

with \(u^{K'}_i = \mathbf{E} : (u^{K,+}_{i,\text{even}}, u^{K,+}_{i,\text{odd}})^T\), where

\[
\begin{align*}
u^{K,+}_{\text{even}} &= \{u^{K,0}_{i,\text{even}}, u^{K,2}_{i,\text{even}}, \ldots, u^{K,2\cdot(K+1)}_{i,\text{even}}\} = \text{CSH}(d^{K'}_i, -\frac{K}{2}) \\
u^{K,+}_{\text{odd}} &= \{u^{K,1}_{i,\text{odd}}, u^{K,3}_{i,\text{odd}}, \ldots, u^{K,2\cdot(K+1)}_{i,\text{odd}}\} = \text{CSH}(d^{K'}_i, 1 - \frac{K}{2})
\end{align*}
\]

The correlation introduced in (2) is translated to the double-length frequency domain by \(d^{K'}_{i+1} = \mathbf{F} \Psi \frac{1}{\sqrt{2}} \mathbf{F}^T d^{K'}_{i+1}\).\]

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Using $\Psi = \text{diag}(e^{-j\frac{2\pi n}{2}}, e^{-j\frac{4\pi n}{2}}, \ldots, e^{-j\frac{2\pi (K+1)n}{2}})$, the $(K+1)$-point DFT matrix $F$, and the interleaving matrix

$$E = \{E_{q,k}\} = \begin{cases}
(1_{K+1}, 0)_{(K+1) \times (K+1)} & \text{for } q = 0, \ldots, 2K; \ k = 0, \ldots, 2K+1 \\
(0, (K+1) \times (K+1))_{1 \times (K+1)} & \text{for } q = 1, 3, \ldots, 2K+1; \ k = 0, \ldots, 2K+1
\end{cases} \quad (6)
$$

Overampled versions for the single-length OFDM symbols $s_i^c$ and $s_{i+1}^c$ are given by

$$s_i^c = \frac{1}{\sqrt{2}} \text{IFFT}(u_i^{c,+}, (K+1)\eta),
$$

$$s_{i+1}^c = \frac{1}{\sqrt{2}} \text{IFFT}(u_i^{c,-}, (K+1)\eta), \quad \eta \in \{0, 1\}
$$

for

$$u_i^{c,+} = \begin{pmatrix}
u_i^{c,+}, u_{i,0}^{c,K/2} & 0_1 \times (\eta-1)(K+1), \\K^{+}_{/2} + u_{i,1}^{c,K+1}, & u_i^{c,-} \end{pmatrix}^T
$$

and

$$u_i^{c,-} = \begin{pmatrix}
u_i^{c,-}, u_{i,0}^{c,K/2} & 0_1 \times (\eta-1)(K+1), \\K^{+}_{/2} - u_{i,1}^{c,K+1}, & u_i^{c,-} \end{pmatrix}^T
$$

The interpolated double-length representation is defined by $s_i^{K\eta} = \eta \text{ IFFT}(u_i^{c,K}, 2\eta(K+1)\eta)$, given zero-padded vector $u_i^{c,K}$. With these results, it is possible to obtain a discrete-time output signal, representing a continuous-time signal with improved continuity properties derived from the SM technique, i.e.,

$$s_i^{SM} = \begin{pmatrix}
u_i^{K\eta}, & s_i^{K\eta} \end{pmatrix} = \begin{pmatrix}
u_i^{K\eta}, & s_i^{K\eta} \end{pmatrix}
$$

where $s_i^{K\eta}$ is expressed in terms of even values for $i$. Let us consider a pair of OFDM symbols $s_i^c$ and $s_{i+1}^c$. Notice that the time instant corresponding to the concatenation point of this pair is now represented by a discrete-time sequence that follows a continuous signal, since it is derived from the double-length representation $s_i^{K\eta}$ in the third row of (9). As a consequence, half of all the original discontinuity points are avoided at this stage, with the consequent reduction in the level of out-of-band power emissions. Note that some latency is introduced, since a joint processing of two consecutive OFDM symbols is required prior to transmission. However, this procedure does not affect the transmission bandwidth.

A. Symbol Extensions

Since a single-tap equalizer is desirable, cyclic extensions are necessary to accommodate the channel dispersion in time domain. In the composition stated for $s_i^{SM}$, we can interpret that cyclic extensions are inserted in a way such that $s_i^c$ and $s_{i+1}^c$ are kept together to allow the validity of (4); so, we can interpret that a CP is inserted before $s_i^c$ to obtain $s_i^{c,CP}$, while a cyclic suffix (CS) is appended to $s_{i+1}^c$ to get $s_{i+1}^{c,CS}$. Meanwhile, when considering the operation of a conventional receiver with standard sampling rate, we suppose that CP extraction is applied on $s_i^{SM}$ considering it as two sequences (i.e., corresponding to two conventional OFDM symbols). Consequently, $s_i^{SM}$ could be obtained from $s_i^{c,CP}$ and $s_i^{c,CS}$ from $s_{i+1}^c$, where $s_{i+1}^{c,CS} = \frac{1}{\sqrt{2}} \text{IFFT}(\Phi_i^{c,-}) = \text{IFFT}(\Phi_i^{c,+})$ for $\Phi_i^{c} = \text{diag}(e^{-j\frac{2\pi n}{2}}, e^{-j\frac{4\pi n}{2}}, \ldots, e^{-j\frac{2\pi (K+1)n}{2}})$.

### B. Multisymbol Extension

We now consider the case where a unique extension is appended to a pair of symbols, to increase the bandwidth efficiency. If only a CS is appended, we name this suffix as multisymbol extension (ME). This approach produces a SM with ME (SMME) signal:

$$s_i^{SMME} = \begin{pmatrix}
u_i^{K\eta}, & s_i^{K\eta} \end{pmatrix} + \begin{pmatrix}
u_i^{K\eta}, & s_i^{K\eta} \end{pmatrix} = \begin{pmatrix}
u_i^{K\eta}, & s_i^{K\eta} \end{pmatrix} \times \begin{pmatrix}
u_i^{K\eta}, & s_i^{K\eta} \end{pmatrix}
$$

IV. SPECTRAL PRECODING

A. Minimal-Order Continuity Constraint

Although the preceding strategy already allows improved spectral properties (with respect to a standard OFDM transmission), in this section we present a complementary spectral precoding technique that can be added for further improvements in system performance. We start by taking (9) into account, emphasizing that the merging strategy has already provided an output signal that avoids half of the original discontinuity points (observed in the concatenation of a conventional sequence of OFDM symbols). Nevertheless, an even better behavior is obtained if we introduce the restriction

$$\frac{dn}{dt} s_i(t) = \frac{dn}{dt} s_{i-1}(t) \quad (11)
$$

to achieve continuity in the signal and its first $N$ derivatives in the union points corresponding to successive $s_i^{SM}$ sequences (with $i = 0, 2, \ldots$). According to the procedure introduced in [12], we start by defining row vector $\rho = \{\rho_n\}$ with entries $(-K/2, -1, 0, 1, \ldots, K/2)$, and matrix $A = (\rho^0, \rho^1, \ldots, \rho^N)^T$ with dimension $(N+1) \times (K+1)$, where superscripts in the elements indicate element-wise powers. To continue, we define two $(K+1) \times (K+1)$ diagonal matrices as $\Phi_1 = \text{diag}(e^{j\phi_0}, e^{j\phi_0}, \ldots, e^{j\phi_0})$ and $\Phi_2 = \text{diag}(e^{j\phi_0}, e^{j\phi_0}, \ldots, e^{j\phi_0})$, for $\phi_0 = 2\pi n$ and $\phi_2 = 2\pi n$. Then, the vector $d_i^{SM} = (d_i^c, d_i^{c,CS})^T$ contains the two OFDM symbols that are merged in frequency domain. Consequently, if $D = ((I_{K+1}, I_{K+1}), (I_{K+1}, -I_{K+1}))^T$ is established, then the $2(N+1) \times 2(K+1)$ matrix defined by

$$B = \begin{pmatrix}
A \Phi_1 & 0_{(K+1) \times (K+1)} \\
0_{(K+1) \times (K+1)} & A \Phi_2
\end{pmatrix}
$$

can express the constraint in (11) as $B \cdot d_i^{SM} = 0_{(N+1) \times 1}$. This restriction holds for OFDM symbols slightly distorted in frequency domain. A distorted symbol is represented as $d_i^{K\eta} = d_i^{K\eta} + w_i$, where the vector $w_i$ with $2(K+1)$ elements identifies the distortion term. Based on this, the precoding matrix can be now written as

$$G = I_{2(K+1)} - B^H(BB^H)^{-1}B. \quad (13)$$
to project the non-distorted vector $d_i^{c'}$ onto the nullspace of $B$ (i.e., $\{x \in \mathbb{C}^{2(K+1)} | Bx = 0_{2(N+1) \times 1}\}$), yielding the distorted vector $\tilde{d}_i^{c''} = Gd_i^{c'}$.

**B. High-Order Continuity Constraints**

A discussion regarding the concatenation points corresponding to $t = \frac{1}{2} T_\text{CP}$ and $t = \frac{3}{4} T_\text{CP}$, implicitly introduced in (9), was presented in [10]. Results therein show that continuity is not strictly guaranteed in these points, although simulation results showed good performance in terms of power spectral density (due to the similarity of the single-length interpolations $s_i^{nK}$ and $s_i^{nK+1}$, and its double-length counterpart $s_i^{3K'}$). However, in the present analysis we report that, for continuity constraints considering the derivatives of $s_i(t)$ (i.e., non-minimal continuity constraints with $N > 0$), the precoder presented in Section IV-A cannot achieve further reductions in the out-of-band power emission. This behavior is derived from the fact that, even though (4) holds for $\eta = 1$, single-length and double-length interpolations are similar in time domain, but their differences are significant when their derivatives are analyzed. Therefore, the robustness of the system improves if additional restrictions are properly defined, taking into account the derivatives of the signal. It was found that the only consideration of the first derivative is enough to guarantee a good performance in terms of out-of-band emissions, independently of the selected value for $N$. This result allows us to keep the low distortion response that was previously presented. In this context, the following $2 \times 2(K+1)$ constraint matrix is defined:

$$B' = \left( \begin{array}{c} A' \Phi_3^{c''} (I_{K+1}, I_{K+1}) \\ A'' \Phi_5^{c''} C' E C D' \end{array} \right),$$

where

$$C = \left( \begin{array}{cc} \text{CSH}(I_{K+1}, \frac{K}{2}) & 0_{(K+1)(K+1)}(I_{K+1}, \frac{K}{2}) \\ 0_{(K+1)(K+1)}(I_{K+1}, \frac{K}{2}) & \text{CSH}(I_{K+1}, \frac{K}{2}) \end{array} \right),$$

$$D' = \left( \begin{array}{cc} (I_{K+1}, 0_{(K+1)(K+1)}) & 0_{(K+1)(K+1)}(I_{K+1}, K) \\ 0_{(K+1)(K+1)}(I_{K+1}, K) & F \Psi_{\frac{1}{K+1}} (F H)^T \end{array} \right),$$

and

$$C' = \text{CSH}(2(I_{K+1}, -(K+1))).$$

To complete this definition, we set $A' = \rho$ and $A'' = -(K+1), \ldots, K$, and we define two diagonal matrices:

$$\Phi_3^{c''} = \text{diag}(e^{j\phi_3 k_0}, e^{j\phi_3 k_1}, \ldots, e^{j\phi_3 k_{K+1}})$$

$$\Phi_5^{c''} = \text{diag}(e^{j\phi_5 k_0}, e^{j\phi_5 k_1}, \ldots, e^{j\phi_5 k_{K+2(K+1)}})$$

with dimension $(K+1) \times (K+1)$ and $2(K+1) \times 2(K+1)$, respectively. Finally, we define $B'' = (B, B')^T$, yielding the precoding matrix

$$G = I_{2(K+1)} - (B'')(H B'')(H)^{-1} B''.$$  

Once the distorted vector $\tilde{d}_i^{c''}$ is obtained, it needs to be split to feed (2) in the symbol merging way (see the Appendix for more details).

**C. Error-Vector Magnitude**

According to [12], the obtained levels of distortion can be evaluated using

$$\text{EVM} = \sqrt{E(\|d_i^{c'} - d_i^{c''}\|^2) / E(\|d_i^{c'}\|^2)}$$

as a generic performance metric. It is important to notice that for the precoder in (13), the latter expression reduces to

$$\text{EVM}_{(N=0)} = \sqrt{(N + 1)/(K + 1)}.$$  

This formula represents a distortion which is significantly lower than the one achieved in [12]. Based on this improvement, higher-order complex modulation schemes can be used. Accordingly, since two new constraints are added for the $N > 0$ case in (17), the EVM expression becomes

$$\text{EVM}_{(N>0)} = \sqrt{(N + 2)/(K + 1)}.$$  

This result indicates that distortion is increased; nevertheless, it still holds a value significantly lower than the one reported in [12]. The observed relationship is represented in Fig. 1 for various $N$. Previous responses hold for the SMME case, too.

**V. Channel Model and Receiver**

Firstly, the channel is modeled as a finite impulse response filter, which remains constant over the duration of a single-length OFDM symbol (i.e., conventional quasi-stationary channel model). Notice that since the proposed system does not affect the communication bandwidth, the double-length representation spans two channel states. Based on this, the following representation holds for $i$ even:

$$\begin{cases} r_{i,CP} = h_i \ast s_i^{C,CP} + n_i \\ r_{i+1,CP} = h_{i+1} \ast s_{i+1}^{C,CS} + n_{i+1} \end{cases},$$

where the channel taps of each state are collected in the vectors $h_i$ and $h_{i+1}$, each one with $\nu(K+1)$ entries. Additive white Gaussian noise (AWGN) is considered, and represented by $n_i$ and $n_{i+1}$, respectively. After CP extraction is performed as in a conventional receiver, $r_i$ and $r_{i+1}$ result.

On a per subcarrier basis, the received vectors in frequency domain can be written as

$$\begin{cases} \tilde{u}_i^{C, +} = \frac{1}{\sqrt{2}} H_i u_i^{C, +} + n_i^\omega \\ \tilde{u}_i^{C, -} = \frac{1}{\sqrt{2}} H_{i+1} Y^{-1} u_i^{C, -} + n_{i+1} \end{cases}$$

for $i$ even, where $H_i$ and $H_{i+1}$ are $(K+1) \times (K+1)$ diagonal matrices containing the channel responses in frequency domain, while $n_i^\omega$ and $n_{i+1}$ are the corresponding frequency domain representations for the noise vectors. Based on this, we conclude that conventional channel estimation in frequency domain is feasible. Then, after CP extraction, fast Fourier
transform processing, and frequency domain equalization are performed in the conventional way, the receiver obtains

\[
\begin{align*}
\hat{u}_{i+k}^{+,+} &= \left(H_i\right)^{-1}F_i \approx u_{i+k}^{+,+} \\
\hat{u}_{i}^{-,+} &= \Upsilon \left(H_{i+1}\right)^{-1}F_{i+1} \approx u_{i}^{-,-}.
\end{align*}
\]

(23)

We highlight that \( \Upsilon \) can be compensated transparently by the pilot-based channel estimation block of the communication system. Maximum likelihood (ML) detection is now applicable in a per subcarrier sense on the 2K symbols of

\[
\begin{align*}
\hat{d}_{i}^{K} &= \text{C强奸} \left(u_{i}^{+,+} + u_{i}^{-,-} - \frac{K}{2}\right) \\
\hat{d}_{i+1}^{K} &= \text{C强奸} \left(u_{i}^{+,+} - u_{i}^{-,-} - \frac{K}{2}\right).
\end{align*}
\]

(24)

Notice that the only non-conventional operations at the receiver side are those specified in (24). Based on this, it is possible to observe that the extra complexity introduced in our proposal is mainly kept at the transmitter side.

A. Symbol Merging with Multisymbol Extension Receiver

In the SMME case, since only \( r_{i+1}^{\text{CP}} \) holds from (21), the sequence is completed by \( \hat{r}_{i} \). Once \( \hat{r}_{i+1} \) is obtained from \( r_{i+1}^{\text{CP}} \), we propose to first estimate \( H_{i+1} \approx u_{i}^{+,+} = \Upsilon F_{i+1} \), and then derive \( u_{i}^{-,-} \) (see (23)), assuming that an adequate pattern of pilot signals is used. With this channel estimation, two vectors that contain the dispersion in time domain can be estimated. The first one is the dispersion from the last elements of \( s_i \) to \( s_{i+1}^{\text{CS}} \), and is called \( z_{i}^{\text{post}} \). The second one is the dispersion from \( s_{i+1}^{\text{CS}} \) to \( s_i \), and is called \( z_{i}^{\text{pre}} \). The processing studied in this work starts with the calculation of \( s_{i+1}^{K} = \text{IFFT} \left(u_{i+1}^{+,+}, K+1\right) \), and subsequently \( s_{i+1}^{K,\text{CP}} \) by inserting the CP. Meanwhile, the channel impulse response \( h_{i+1} \) can be derived from \( H_{i+1} \), and the \((1+2v) (K+1)\)-element convolution vector \( y_i = h_{i+1} \ast s_{i+1}^{K,\text{CP}} \) is used to obtain

\[
\begin{align*}
\hat{z}_{i}^{\text{pre}} &= \left((y_i(1+(1+2v)(K+1)+1), y_i(1+(1+2v)(K+1)+2), \ldots, y_i(1+(1+2v)(K+1))\right) \\
\hat{z}_{i}^{\text{post}} &= \left(\hat{r}_{i} \ast y_i(0), \hat{r}_{i} \ast y_i(1), \ldots, \hat{r}_{i} \ast y_i(v(K+1)+1)\right) \cdot (K+1)\times 1.
\end{align*}
\]

(25)

After that, \( \hat{u}_{i+k}^{+,+} \) can be estimated by the aid of \( \hat{u}_{i}^{+,+} = 1 \hat{r}_{i} - \hat{z}_{i}^{\text{pre}} + \hat{z}_{i}^{\text{post}} \). Once \( H_{i} \approx H_{i} \) is estimated based on pilot signals, \( \hat{u}_{i+k}^{+,+} \) can be obtained and ML detection can be applied as in (24).

VI. SIMULATION RESULTS

First of all, it is important to say few words about the peak-to-average power ratio (PAPR) behavior of our proposed technique. Computer simulations showed that our approach does not amplify the PAPR, despite the increased-length IFFT operation that is implicit in (9) and (10). For brevity, these simulation results are not included in this paper.

A. Power Spectra

Our proposal was tested by means of numerical simulations carried out for an OFDM system with \( K = 600 \) subcarriers that employ 16-QAM modulation. The rest of the OFDM settings and the power spectra estimation method follow the ones specified in [7]. In Fig. 2, the performance of this conventional OFDM system compliant with subcarrier allocation scheme (1) (and precoded according to [12]) is represented by dotted lines, and compared with the equivalent SM scheme proposed in this work, which is indicated by dashed lines. In addition, the performance of the SMME proposal is presented using continuous lines.

A constant gap of 3 dB is observed when analyzing the reduction in the level of out-of-band power emissions for the SM scheme. This gap is enhanced to 6 dB for the SMME configuration, when SP is used. Note that an LTE mask is also presented in Fig. 2. The reduction gap that our proposal provides (when compared to the conventional case and scheme presented in [12]) allows to achieve the out-of-band emission requirement with \( N = 1 \), while [12] requires \( N = 2 \). This illustrative case shows the way in which our proposal reduces the undesired power emissions, providing at the same time a response with lower levels of distortion (when compared to other alternatives). In other words, given a target mask, the SM or SMME schemes allow to mitigate out-of-band power emission; this enables the precoder to choose lower continuity orders, which in turn reduce the level of distortion. The observed reduction patterns have the novel feature of achieving a constant reduction gap. This capability reduces considerably the interference power that is induced in adjacent channels (occupied by neighboring systems), with an important diminution in the region near (but external) to the limits of the frequency band assigned for the SM/SMME signal. Note that this feature is difficult to implement in practice using conventional filtering techniques.

B. Bit Error Rate

In this section, we first consider an AWGN channel, by assuming that \( n_i \) is a complex-valued zero-mean Gaussian noise vector with covariance matrix \( \sigma_n^2 \mathbf{I} \), and \( H_i = \mathbf{I}^{K+1} \). After that, a frequency-selective fading channel is analyzed, and represented by means of a complex-valued diagonal matrix \( H_i \), corresponding to an \( h_i \) vector with \( v(K+1)+1 \) elements (i.e., an stochastic vector with zero-mean unit variance complex Gaussian entries). Figure 3 shows the performance obtained in both cases. An equivalent bit error rate (BER) response is observed in presence of an AWGN channel at low \( E_b/N_0 \) conditions, when comparing the performance of a conventional OFDM system with our proposal. In case of Rayleigh fading, a slight penalization is noticed when the system works in a noise.
limited regime. However, it is possible to observe that this undesired effect reduces gradually as the $E_b/N_0$ in the system increases. In addition, a lower error floor can be achieved for high distortion cases (derived from higher values of $N$) in both, AWGN and Rayleigh channels.

The SMME configuration was tested in a Rayleigh fading channel that follows the Jakes’ model with a Doppler frequency of 50 Hz (i.e., receiver speed close to 40 km/h for a 2.6 GHz carrier frequency). To produce the curves that appear in Fig. 4, the duration of the CS was set to 1.77 $\mu$s, and a constant channel delay profile with 16-tap was used. A pilot insertion ratio equal to eight was used, and channel estimation was implemented based on the least-squared method. Dashed lines present the performance of conventional OFDM system with SP [12]. Continuous lines show the strongly improved robustness of the SMME approach, attaining gains of 17 dB for BER = $6.10^{-3}$ and QPSK. Lower error rate floors can be achieved using more advanced channel estimation methods.

VII. CONCLUSIONS

A novel framework to generate an interpolated digital signal for OFDM transmissions was presented. The key idea behind this approach was to correlate two consecutive OFDM symbols in frequency domain. The use of a double-length inverse discrete Fourier transform (IDFT) calculation allows an important reduction in the out-of-band power emissions, provided a convenient vectorial conformation of the output signal (based on single-length and double-length interpolations) is used, and a low distortion spectral precoding technique is employed. The proposed scheme does not require additional bandwidth or sampling rate, and represents a useful approach that can be easily applied in OFDM transmissions with high restrictions in out-of-band power emissions. Since the signal is digitally generated, hardware reconfiguration capabilities are able to implement this approach in a robust way.

APPENDIX

SP CONSTRAINT DEFINITION FOR SMME

Firstly, we need to redefine some matrices. We start with (12), i.e., $B = A\Phi_2 \cdot (I_{K+1}, -I_{K+1})$. Then, if $\rho' = \{\rho'_k\}$ with entries $(-(K+1), \ldots, -1, 0, 1, \ldots, K)$, we can define

\[ A'' = \frac{1}{2} (\rho^0, \rho^1, \ldots, \rho^N) \]

which is a row-extended version of $A''$. To allow the precoding matrix $G$ in (17) be based on $B''$ we reformulate (14) as

\[ B'' = (A' \Phi_3 (I_{K+1}, I_{K+1}), A'' \Phi_3 C'ECD, A'''C'ECD')^T. \]

REFERENCES